2D PRODUCTION MODEL WITH ENVIRONMENTAL FACTORS

Michael Dymkov[1], Alex Brilevskii[2] and Siarhei Dymkov[3]

[1]Belarus State Economic University, Minsk, Belarus
[2]Bread-making Company, Minsk, Belarus
[3]Belarus Institute of Labour and Social Relations, Minsk, Belarus
[1]*dymkov_m@bseu.by*, [2]*brilevski@yandex.by*, [3]*dymkov78@mail.ru*

Multistep 2D dynamical systems arise in mathematical modeling of various industrial and economic processes. Examples of such industrial productions give chemical-technological processes, complex high-tech production lines conveyor type, cascading treatment facilities, large multi-disciplinary and inter-related production in economy, and others.

Below model of economic system that takes into account the environmental factor, when along with the process of production of goods there is the destruction of damage to the environment posed by the production process. The main difficulty of the considered mathematical models associated with the natural requirement of nonnegativity of the parameters and variables involved.

In this paper to investigate the multistage positive systems we use an operator approach developed in [1]. Some results on the theory of positive 1D and 2D systems can be found in [2], a new method of studying non-negative systems of linear equations and their operator counterparts, which is used in the description of economical models, outlined in [3].

At present, the task of reducing damage of hazardous waste to the environment has become of paramount importance to all economic activity. Therefore becoming increasingly important mathematical models of economic systems, which along with the process of production of goods there is the destruction of environmental damage arising from this production. It seems, the first time this type of model were considered by W. Leontief in [4].

Suppose now that:

1) there is a set of products (goods) consumed by consumers and for the production of which is required to implement technological cycle consisting of successive $s = 1, 2, ..., h$ steps (or stages) of processing, i.e. product is considered ready, if implemented all h prescribed stages;

2) production of goods is planned for a long time, so that after completion of the current k-th ($k = 1, 2, \ldots$) cycle, the production process is resumed on the next $(k+1)$ cycle. Long term means that the case where $k \to \infty$ is possible;

3) at the next $(s+1)$ stage of the new $(k+1)$ cycle the goods produced at the previous s stage of the current $(k+1)$ cycle are used for production of new goods, the destruction of demages occurred in the previous cycle k, and for consumption on current ($k+1$) cycle;

4) damages, arising from such proceedings in s - stage of current $(k+1)$ cycle, in turn, consist, first, from the damage received from the production benefits of the current $(k+1)$ cycle, and secondly, from the destruction of damage from the previous k-th cycle, obtained by s-th stage, and third, from the damage remaining in the environment;

5) a set of benefits and damages consist of n and m pieces of items, respectively.

For the mathematical description of such production, we introduce the necessary definitions and notations.

For each pair (k, s), $s = 1, 2, ..., h$; $k = 1, 2, \ldots$, denote by:

1) $x(k,s), = \left(x_1(k,s), x_2(k,s), ..., x_n(k,s)\right) \in \mathrm{R}^n$ --- the volume of goods being to the beginning s stage of k - th production cycle;

2) $y(k,s),=(y_1(k,s),y_2(k,s),...,y_m(k,s))\in R^m$ --- the amount of damage occurring in the production process and destroyed by the system at the beginning s stage of k - th production cycle;

3) $c(k,s),=(c_1(k,s),c_2(k,s),...,c_n(k,s))\in R^n$ --- the amount of goods consumed in the s-th stage in the k-th cicle;

4) $d(k,s)=(d_1(k,s),d_2(k,s),...,d_m(k,s))\in R^m$ --- the amount of damage remaining in the environment caused by production at s-th stage in the k-th cycle;

5) a_{il} --- share of l-th type of benefits needed to produce a unit of the good i-th type $(i=1,\dots,n;\quad j=1,\dots,m)$;

B_{ij}^0 --- share of manufactured goods i-th type used in the process of destroying damages j-th type $(i=1,\dots,n;\quad j=1,\dots,m)$;

C_{jl} --- the share of damage j - th type, arising in the production of goods l-th type $(l=1,\dots,n;\quad j=1,\dots,m)$;

D_{qj}^0 --- the share of damages j-th type, resulting in the destruction of damages q - th type $(j=1,\dots,m;\ q=1,\dots,m)$;

From the physical meaning of the problem that with all of the variables and coefficients are nonnegative.

Then, using the notation dynamics of the balance amount of benefits and losses of production is described by the equations

$$\begin{aligned} x_1(k+1,s+1) &= a_{11}x_1(k+1,s)+\ldots+a_{1n}x_n(k+1,s)\\ &+ b_{11}^0 y_1(k,s)+\ldots+b_{1m}^0 y_m(k,s)+c_1(k+1,s),\\ &\ldots\ \ldots\ldots\ldots\\ x_n(k+1,s+1) &= a_{n1}x_1(k+1,s)+\ldots+a_{nn}x_n(k+1,s)\\ &+ b_{n1}^0 y_1(k,s)+\ldots+b_{nm}^0 y_m(k,s)+c_n(k+1,s), \end{aligned} \tag{1}$$

$$\begin{aligned} y_1(k+1,s) &= c_{11}x_1(k+1,s)+\ldots+c_{1n}x_n(k+1,s)+\\ &+ d_{11}^0 y_1(k,s)+\ldots+d_{1m}^0 y_m(k,s)-d_1(k+1,s),\\ &\ldots\ \ldots\ldots\ldots\\ y_m(k+1,s) &= c_{m1}x_1(k+1,s)+\ldots+c_{mn}x_n(k+1,s)+\\ &+ d_{m1}^0 y_1(k,s)+\ldots+d_{mm}^0 y_m(k,s)-d_m(k+1,s) \end{aligned} \tag{2}$$

It should be noted, that the lag (shift) for the argument k in (1) □ (2) for the damage variables looks natural, because, for example, it takes time for the delivery of relevant products to the place, or the shift can be set technological standards.

Denote

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ .. & \dots & \dots & \dots \\ a_{n1} & a_{n2} & .. & a_{nn} \end{pmatrix}, \quad B_0=\begin{pmatrix} b^0_{11} & b^0_{12} & \dots & b^0_{1m} \\ .. & \dots & \dots & \dots \\ b^0_{n1} & b^0_{n2} & .. & b^0_{nm} \end{pmatrix},$$

$$C=\begin{pmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ .. & \dots & \dots & \dots \\ c_{m1} & c_{m2} & .. & c_{mn} \end{pmatrix}, \quad D_0=\begin{pmatrix} d^0_{11} & d^0_{12} & \dots & d^0_{1m} \\ .. & \dots & \dots & \dots \\ b^0_{m1} & b^0_{m2} & .. & b^0_{mm} \end{pmatrix}. \tag{3}$$

Tus, the multistage multisectoral production model with ecological factors is given as

$$\begin{cases} x(k+1,s+1)=Ax(k+1,s)+B_0y(k,s)+c(k,s), \\ y(k+1,s)=Cx(k+1,s)+D_0y(k,s)-d(k,s). \end{cases} \tag{4}$$

where

$$A\in \mathrm{R}^{n\times n}_+,\ B_0\in \mathrm{R}^{n\times m}_+,\ C\in \mathrm{R}^{m\times n}_+,\ D_0\in \mathrm{R}^{m\times m}_+,$$
$$x(k,s)\ge 0,\ y(k,s)\ge 0,\ c(k,s)\ge 0,\ d(k,s)\ge 0,\ \ \forall (k,s). \tag{5}$$

The initial data for (4) are

$$x(k,0)=\alpha_{k+1},\quad k\ge 0; y(0,s)=f_s,\quad s=1,2,\dots,h. \tag{6}$$

The difficulty that arises in the study of the mathematical model associated with the requirement of nonnegativity of the parameters and variables involved.

Note, that by introducting the block matrices (operators) and extended vectors (in the space of sequences) these models in principle can be reduced to a generalized equation, similar in form to the equation of open-economyLeont'ev model

$$z=\mathrm{A}z+w.$$

However, this reduction is inefficient, because here the main obstacle lies in the fact that, unlike the model of Leontief the vector w of the components corresponding to the production of goods, non-negative, and the portion of injury nonpositive, although the vector z is nonnegative.

One of the problems under consideration is to describe the conditions on the technological matrix A, B_0, C, D_0, in which the system of equations (4)-(6) has a nonnegative solution (x,y,d) at any level of demand for c (conditions of productivity technological matrices).

Another ptroblem is to examine the possibility for a given level of demand c to reduce the amount of damage remaining in the environment, i.e. whether the system (4)-(6) for a given level of demand c a nonnegative solution of the form $(x,y,0)$ (conditions of compensability).

References

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